International Journal of Computer Science Engineering
and Information Technology Research (IJCSEITR)
ISSN(P): 2249-6831; ISSN(E): 2249-7943
Vol. 4, Issue 3, Jun 2014, 15-24

FUZZY TOPSIS AND FUZZY VIKOR METHODS USING THE TRIANGULAR FUZZY HESITANT SETS

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ABSTRACT

The decision maker needs to choose the most suitable alternative from the available alternatives. This study mainly focuses on two multiple criteria decision making (MCDM) methods TOPSIS and VIKOR Method in the context of recently developed hesitant fuzzy theory. Both the methods are based on an aggregating function that represents closeness to the ideal solution. A solution obtained by fuzzy TOPSIS method has the shortest distance from the ideal one and farthest from the negative ideal solution. Fuzzy VIKOR method helps to determine a compromise solution that gives a maximum group utility for the majority and minimum for opponents. Hesitant Fuzzy Sets (HFSs) are very helpful in decision making situations when a decision maker might consider different degrees of membership. In this paper in order to solve MCDM problems the fuzzy TOPSIS and Fuzzy VIKOR methods are developed usingTriangular Fuzzy Hesitant Fuzzy Sets. A procedure using both the methods is presented. Finally a numerical example illustrates the applicability of the proposed methods and the results are also compared.

KEYWORDS: Fuzzy VIKOR, Fuzzy TOPSIS, Multiple Criteria Decision Making, Triangular Fuzzy Hesitant Fuzzy Sets

1. INTRODUCTION

Fuzzy set theory has been extensively and effectively applied in many different areas to handle uncertainty in many decision making situations. When different sources of vagueness appear simultaneously it has some restrictions to deal with imprecise and vague information.In order to overcome such limitations, different extensions of fuzzy sets (FS) have been introduced in the literature such as(i)Atanassov's intuitionistic fuzzy sets (IFS) [1] which considers simultaneously the membership degree and the non-membership degree of each element, (ii) type-2 fuzzy sets (T2FS) [2] that use a fuzzy set over the unit interval to model uncertainty, (iii) interval-valued fuzzy sets (IVFS) [3, 4] in which the membership degree of an element is given by a closed subinterval of the unit interval(iv) fuzzy multisets [5] where the membership degree of each element is given by a subset of [0,1], and so on.In establishing the membership degree of an element the common difficulty that often appears is that there are some possible values that make the decision maker (DM) to hesitate to choose among many values. To overcome this difficultyand to manage simultaneous sources of vagueness, Torra [6] introduced a new extension of fuzzy sets called Hesitant Fuzzy Sets (HFSs)which is very helpful indecision making situations when an expert might consider different degrees of membership. Sometimes, it is difficult for DMs to express the membership degrees of an element to a given set only by crisp values between 0 and 1. In order to model this

hesitation, Yu [7] introduced the concept of Triangular Fuzzy Hesitant Fuzzy Set (TFHFS), whose membership degrees of an element to a fuzzy set are expressed by several triangular fuzzy numbers [8].

The VIKOR method is a multicriteria decision making (MCDM) method. It was originally developed by Serafim Opricovic to solve decision problems with conflicting and non-commensurable criteria, assuming that compromise is acceptable for conflict resolution. VIKOR ranks alternatives and determines the compromise solution closest to the ideal solution. Po-Lung Yu in 1973[9] and by Milan Zeleny [10] introduced the idea of compromise solution in MCDM. The name VIKOR appeared in 1990 [11] from Serbian: VIse Kriterijumska Optimizacija I Kompromisno Resenje, that means: Multicriteria Optimization and Compromise Solution, with pronunciation: VIKOR. The international recognition of the VIKOR method was due to contribution of Serafim Opricovic and Gwo-Hshiung Tzeng (2004) [12]. In this study the Fuzzy VIKOR method has been developed to solve problem in a triangular hesitant fuzzy environment. The triangular fuzzy numbers are used to handle imprecise numerical quantities. Fuzzy VIKOR is based on the aggregating fuzzy merit that represents distance of an alternative to the ideal solution.

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) is another popular approach to MCDM. The main idea is that the best alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The TOPSIS method has beenextended to deal with TFHFS. A comparative study on two MCDM methods namely fuzzy VIKOR and fuzzy TOPSIS under hesitant fuzzy environment is presented. A comparative analysis is illustrated with a numerical example.

The paper is organized as follows. Section 2 introduces the concept of HFS and TFHFS and their basic operations. Section 3 presents the procedure for solving MCDM using Fuzzy VIKOR Method based on TFHFS. Sections 4 develops a fuzzy TOPSIS method using TFHFS. In Section 5 a numerical example illustrates the applications of these methods undertriangular hesitant fuzzy environment.

2. BASIC CONCEPTS

The concept of hesitant fuzzy sets was first introduced by Torra V, Narukawa Y [13]

2.1 **Definition [13]**

Let X be a fixed set, an Hesitant Fuzzy Set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1]. Mathematically Xia and Xu express the HFS as, $H = \{\langle x, h_H(x)\rangle / x \in X\}$ where $h_H(x)$ is a set of some values in [0, 1] denoting the possible membership degrees of the element $x \in X$ to the set H. $h_H(x)$ is called the hesitant fuzzy element (HFE)

2.2 **Operations on HFEs [14]**

Let h, h_1 *and* h_2 be three HFEs. Then

- $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$
- $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$

- $h^{\lambda} = \bigcup_{\gamma \in h} \{\gamma^{\lambda}\}$
- $\lambda h = \bigcup_{\gamma \in h} \{1-(1-\gamma_1)^{\lambda}\}, \quad \lambda > 0$

Xia and Xu defined the score function to compare two HFEs as below.

2.3 Definition [14]: Score Function

Let h be an HFE. Then $s(h) = \frac{1}{n(h)} \sum_{\gamma \in h} \gamma$ is called the score function of h, where n (h) is the number of values of h. If $s(h_1) > s(h_2)$ then $h_1 > h_2$ and if $s(h_1) = s(h_2)$ then $h_1 = h_2$

2.4 Definition [15]

A fuzzy number a on R is called a triangular fuzzy number if its membership function $\mu_a : R \to [0,1]$ are equal to

$$\mu_a(x) \begin{cases} \frac{x-a^L}{a^M - a^L}, & x \in [a^L, a^M] \\ \frac{x-a^U}{a^M - a^U}, & x \in [a^M, a^U] \\ 0 \quad , & otherwise \end{cases}$$

The concept of Triangular Fuzzy Hesitant fuzzy sets (TFHFS) [7] was introduced because of the fact that mostly precise membership degrees of an element to a set are impossible. In THHFS membership degrees of an element to a set are represented by several possible triangular fuzzy numbers.

2.5 **Definition [7]**

Let X be a fixed set, a Triangular Fuzzy Hesitant Fuzzy Set $(TFHFS)E$ on X is in terms of a function $f_E(x)$ that returns several triangular fuzzy values, denoted by $E = \{< x, f_E(x) > / x \in X\}$ where $f_{\bar{E}}(x)$ is a set of several triangular fuzzy numbers, expressing the possible membership degrees of the element $x \in X$ to the set E

If $f_E(x_i) = \{(\xi^L, \xi^M, \xi^U) / \xi \in f_E(x_i)\}$ then $f_E(x_i)$ is called the Triangular Fuzzy Hesitant element (TFHFE). Here ξ^L, ξ^M and ξ^U stand for lower, modal and upper values respectively of the support f

The TFHFS defined in terms of a function when applied to $x \in X$ returns several triangular fuzzy numbers of R. Thus it is an extension of HFS.

2.6 Operations on TFHFES [7]

Let f , f_1 and f_2 be three TFHFEs. Then,

- $f_1 + f_2 = \left\{ \left(\xi_1^L + \xi_2^L - \xi_1^L \xi_2^L, \xi_1^M + \xi_2^M - \xi_1^M \xi_2^M, \xi_1^U + \xi_2^U - \xi_1^U \xi_2^U \right) / \xi_1 \in f_1, \xi_2 \in f_2 \right\}$

- $f_1 \otimes f_2 = \left\{ \left(\xi_1^L . \xi_2^L, \xi_1^M . \xi_2^M, \xi_1^U . \xi_2^U \right) / \xi_1 \in f_1, \xi_2 \in f_2 \right\}$

- $f^{\lambda} = \left\{ \left(\xi_1^L \right)^{\lambda}, \left(\xi_1^M \right)^{\lambda} \left(\xi_1^U \right)^{\lambda} / \xi_1 \in f, \xi_2 \in f_2 \right\}, \ \lambda > 0$

- $\lambda f = \left\{ \left(1 - \left(1 - \xi_1^L \right)^{\lambda}, 1 - \left(1 - \xi_1^M \right)^{\lambda}, 1 - \left(1 - \xi_1^U \right)^{\lambda} \right) \right\}, \ \lambda > 0$

We define the hesitant normalized Hamming distance measure between two TFHFS f_1 and f_2 as follows:

2.7 Definition

Let f_1 and f_2 be two TFHFS on $X = \{x_1, x_2, ..., x_n\}$ then the hesitant normalized Hamming distance measure between f_1 and f_2 is defined as

$\left\| f_1 - f_2 \right\| = \frac{1}{l} \sum_{j=1}^{l} \left(\left| f_{1\sigma_j}^L, - f_{2\sigma_j}^L \right| + \left| f_{1\sigma_j}^M - f_{2\sigma_j}^M \right| + \left| f_{1\sigma_j}^U - f_{2\sigma_j}^U \right| \right)$ where $l(f)$ is the number of elements is TFHFS f and $l = \max\left(l(f_1), l(f_2) \right)$. If $l(f_1) \neq l(f_2)$, then the minimal TFHFS is extended until both *TFHFS* f_1 and f_2 have the same length so that it can be compared. The selection of the triangular fuzzy value to be added depends on the risk preferences of the decision makers. The optimist adds the maximum value while pessimists add the minimum value to equalize the lengths of TFHFS.

3. FUZZY VIKOR METHOD USING TFHFS

The Vikor method is used for decision making with TFHFS

Step-1: Determine the positive triangular ideal solution (PTIS) and the negative triangular ideal solution (NTIS).

$A^+ = \left\{ f_1^+, f_2^+, ..., f_n^+ \right\}$ where

$$f_j^+ = \bigcup_{i=1}^{m} f_{ij} = \bigcup_{\gamma_{1j} \in f_{1j}, ... \gamma_{mj} \in f_{mj}} \left(\max\left(\gamma_{1j}^L, ..., \gamma_{mj}^L \right), \max\left(\gamma_{1j}^M, ..., \gamma_{mj}^M \right), \max\left(\gamma_{1j}^U, ..., \gamma_{mj}^U \right) \right)$$

$A^- = \left\{ f_1^-, f_2^-, ..., f_n^- \right\}$ where $f_j^- = \bigcap_{i=1}^{m} f_{ij}$

$$=\bigcup_{\gamma_{1j}\in f_{1j},\ldots\gamma_{mj}\in f_{mj}}\left(\min\left(\gamma_{1j}^{L},\ldots,\gamma_{mj}^{L}\right),\min\left(\gamma_{1j}^{M},\ldots,\gamma_{mj}^{M}\right),\min\left(\gamma_{1j}^{U},\ldots,\gamma_{mj}^{U}\right)\right)\left(j=1,2,\ldots,n\right)$$

Step-2: Compute S_i and R_i as below

$$S_i=\sum_{j=1}^{n}\frac{\omega_j\left\|f_j^{+}-f_{ij}\right\|}{\left\|f_j^{+}-f_j^{-}\right\|},\left(i=1,2,\ldots,m\right)\text{ and } R_i=\max_j\frac{\omega_j\left\|f_j^{+}-f_{ij}\right\|}{\left\|f_j^{+}-f_j^{-}\right\|}\left(i=1,2,\ldots,m\right)$$

where ω_j are the weights of the criteria expressing their relative importance.

Step-3: Compute the values $Q_i,\left(i=1,2,,\ldots\text{m}\right)$

$$Q_i=v\left(\frac{S_i-S^{+}}{S^{-}-S^{+}}\right)+\left(1-v\right)\left(\frac{R_i-R^{+}}{R^{-}-R^{+}}\right)\text{where } S^{+}=\min_i S_i,\quad S^{-}=\max_i S_i\,;$$

$R^{+}=\min_i R_i,\ \ R^{-}=\max_i R_i$ and 'v' is introduced as weight of the strategy of the maximum group utility.

Step-4: Rank the alternatives by sorting the values of S, R and Q in decreasing order which results in three ranking lists.

Step-5: Propose as a compromise solution the alternative $A^{'}$ which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

C1: Acceptable advantage: $Q\left(A''\right)-Q\left(A^{'}\right)\geq DQ,$ where A'' is the alternative with second position in the ranking list by Q; DQ=1/(m-1) where m is the number of alternatives.

C2: Acceptable stability in decision: Alternative $A^{''}$ must also be the best ranked by D or/and R. This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when v > 0.5 is needed) or by "consensus" $v\approx 0.5$ or with" veto" $\left(v<0.5\right)$. Here v is the weight of the decision making strategy "the majority criteria" or ("maximum group utility"). If one of the two conditions is not satisfied, then a set of compromise solutions is proposed, which consists of

- Alternatives A'' and A' if only condition C2is not satisfied, or
- Alternatives A', A"… $A^{(m)}$ if condition C1 is not satisfied, $A^{(m)}$ is determined by the relation $Q\left(A^{(m)}\right)-Q\left(A^{''}\right)<DQ$ for maximum m (the positions of these alternatives are "in closeness").

4. FUZZY TOPSIS METHOD FOR THE MULTI CRITERIA DECISION MAKING USING TFHFS

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), one of the known classical MCDM methods was first developed by Hwang and Yoon [16].The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Now a procedure for Fuzzy TOPSIS Method using TFHFS is presented.

Step-1: Determine the ideal and negative-ideal solution

$$A^{+}=\left\{f_1^{+},..,f_n^{+}\right\} \text{ where } f_j^{+}=\bigcup_{i=1}^{m} f_{ij}=\bigcup_{\gamma_{ij}\in f_{ij},...,\gamma_{mj}\in f_{mj}} \max\left(\gamma_{1j},...,\gamma_{mj}\right) j=1,2,...,n$$

$$A^{-}=\left\{f_1^{-},..,f_n^{-}\right\} \text{ where } f_j^{-}=\bigcup_{i=1}^{m} f_{ij}=\bigcup_{\gamma_{ij}\in f_{ij},...,\gamma_{mj}\in f_{mj}} \min\left(\gamma_{1j},...,\gamma_{mj}\right) j=1,2,...,n$$

Step-2: Calculate the separation measures, using the hesitant normalized Hamming distance. The separation of each alternative from the ideal solution is given as $D_i^{+}=\left\|A_i-A^{+}\right\|=\sum_{j=1}^{n}\left\|f_{ij}-f_j^{+}\right\|$

Similarly, the separation from the negative ideal solution is given as $D_i^{-}=\left\|A_i-A^{-}\right\|=\sum_{j=1}^{n}\left\|f_{ij}-f_j^{-}\right\|$

Step-3: Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^{+} is defined as $D_i=\dfrac{D_i^{-}}{D_i^{-}+D_i^{+}}$

Step-4: Rank the preference order and the alternative with the largest relative closeness is chosen as the best alternative.

5. NUMERICAL EXAMPLE

A numerical example of teacher Quality evaluation adopted from Yu [7] is taken for this comparative study. The teaching work of the four lecturers Ai (i = 1, 2, 3,4) are mainly evaluated based on the Teaching effect C1, teaching method C2, teaching plan C3 and teaching content C4. Suppose the important degree of the attributes is ω = (0.17, 0.35, 0.26, 0.22) ^{T}and the decision matrix is presented in the form of triangular fuzzy hesitant fuzzy values as in Table 1.

Table 1: Triangular Fuzzy Hesitant Fuzzy Decision Matrix

	G_1	G_2	G_3	G_3
A_1	{(0.3, 0.4, 0.5)}	{(0.4, 0.5, 0.6)(0.7,0.8,0.9)}	(0.5, 0.7, 0.7)(0.7,0.8,0.9)}	{(0.2, 0.3, 0.4)(0.3,0.4,0.5)}
A_2	{(0.1, 0.2, 0.3)(0.2,0.3,0.4)}	{(0.5, 0.6, 0.7)(0.3,0.4,0.5)}	{(0.2, 0.4, 0.6)(0.7,0.8,0.9)}	{(0.1, 0.4, 0.7)(0.6,0.7,0.8)}
A_3	{(0.1,0.2, 0.3)(0.3,0.4,0.5)}	{(0.7, 0.8, 0.9)}	{(0.2, 0.3, 0.4)(0.5,0.6,0.7)}	{(0.4, 0.5, 0.6)}
A_4	{(0.2, 0.3, 0.4)}	{(0.3, 0.4, 0.5)(0.2,0.4,0.6)}	{(0.5, 0.6, 0.7)(0.3,0.4,0.5)}	{(0.1, 0.2, 0.3)(0.3,0.4,0.5)}

5.1 Fuzzy VIKOR Method Using TFHFS

The procedure for VIKOR method consists of the following steps.

Step-1: Determine the positive ideal solution (PIS) and negative ideal solution (NIS)

$$A^{+}=\begin{Bmatrix} f_1^{+} & f_2^{+} & f_3^{+} & f_4^{+} \\ (0.3,0.4,0.5) & (0.7,0.8,0.9) & (0.7,0.8,0.9) & (0.6,0.7,0.8) \end{Bmatrix}$$

$$A^- = \begin{Bmatrix} f_1^- & f_2^- & f_3^- & f_4^- \\ (0.1, 0.2, 0.3) & (0.2, 0.4, 0.5) & (0.2, 0.3, 0.4) & (0.1, 0.3, 0.4) \end{Bmatrix}$$

Step-2: Compute S_i and R_i as below

$$S_i = \sum_{j=1}^{n} \frac{\omega_j \left\| f_j^+ - f_{ij} \right\|}{\left\| f_j^+ - f_j^- \right\|}, (i = 1, 2, ..., m) \text{ and } R_i = \max_j \frac{\omega_j \left\| f_j^+ - f_{ij} \right\|}{\left\| f_j^+ - f_j^- \right\|} (i = 1, 2, ..., m)$$

where ω_j are the weights of the criteria expressing their relative importance.

$$S_1 = \frac{\omega_1 \left\| f_1^+ - f_{11} \right\|}{\left\| f_1^+ - f_1 \right\|} + \frac{\omega_2 \left\| f_2 - f_{12} \right\|}{f_2^+ - f_2^-} + \frac{\omega_3 \left\| f_3^+ - f_{13} \right\|}{f_3^+ - f_3^-} + \frac{\omega_4 \left\| f_4^+ - f_{14} \right\|}{f_4^+ - f_4^-}$$

S_1= 0.3182 and R_1 = 0.154

$$S_2 = \frac{\omega_1 \left\| f_1^+ - f_{21} \right\|}{\left\| f_1^+ - f_1 \right\|} + \frac{\omega_2 \left\| f_2 - f_{22} \right\|}{f_2^+ - f_2^-} + \frac{\omega_3 \left\| f_3^+ - f_{23} \right\|}{f_3^+ - f_3^-} + \frac{\omega_4 \left\| f_4^+ - f_{24} \right\|}{f_4^+ - f_4^-}$$

S_2 = 0.5075 and R_2 = 0.21

Similarly S_3 = 0.355andR3 = 0.182

S_4 = 0.5695 and R_4 = 0.3231

Step-3: Let v = 0.5. Compute the values of Q_i using the formula

$$Q_i = \frac{v\ (S_i - S^+)}{S^- - S^+} + (1 - v) \frac{R_i - R^+}{R^- - R^+}$$

$$S^+ = \min S_i = \ 0.3182 \ R^+ \ = \min R_i = \ 0.154$$

$$S^- = \max S_i = \ 0.5695 \text{, } R^- = \max R_i = \ 0.3231$$

$$Q_1 = 0 \text{; } Q_2 = 0.5422 \text{ ; } Q_3 = \ 0.1560 \text{; } Q_4 = 1.0$$

Step-4: Rank the alternatives, sorting by the values S, R and Q in decreasing order which results in three ranking lists as shown in Table 2

Table 2: The Ranking of the Alternatives

	A_1	A_2	A_3	A_4	Ranking
S	0.3182	0.5075	0.355	0.5847	$A_1 > A_3 > A_2 > A_4$
R	0.154	0.21	0.182	0.3231	$A_1 > A_3 > A_2 > A_4$
Q	0.4337	0.5422	0.1560	1.0	$A_1 > A_3 > A_2 > A_4$

By ranking the preference order and by compromise solution A_1 is the best alternative.

From table 3 we find that the values of Q_2 increases as the value of weight v increases, the values of Q_3 decreases as the value of weight v increases while values of Q_1 and Q_4 remains unaltered. The ranking of the alternatives is not affected by the change of weights.

Table 3: The Ranking of the Alternatives Based on the Values of Q as Weight v Changes

v	A_1	A_2	A_3	A_4	Ranking
0	0	0.3312	0.1656	1	$A_1 > A_3 > A_2 > A_4$
0.1	0	0.3734	0.1637	1	$A_1 > A_3 > A_2 > A_4$
0.2	0	0.4156	0.1618	1	$A_1 > A_3 > A_2 > A_4$
0.3	0	0.4578	0.1598	1	$A_1 > A_3 > A_2 > A_4$
0.4	0	0.5000	0.1579	1	$A_1 > A_3 > A_2 > A_4$
0.5	0	0.5422	0.1560	1	$A_1 > A_3 > A_2 > A_4$
0.6	0	0.5844	0.1541	1	$A_1 > A_3 > A_2 > A_4$
0.7	0	0.6266	0.1522	1	$A_1 > A_3 > A_2 > A_4$
0.8	0	0.6689	0.1503	1	$A_1 > A_3 > A_2 > A_4$
0.9	0	0.7111	0.1484	1	$A_1 > A_3 > A_2 > A_4$
1.0	0	0.7533	0.1464	1	$A_1 > A_3 > A_2 > A_4$

5.2 Fuzzy TOPSIS Method Using TFHFS

Now the procedure for hesitant fuzzy TOPSIS method is as follows.

Step-1: Determine the ideal and non-negative ideal solution

$$A^+ = \begin{Bmatrix} f_1^+ & f_2^+ & f_3^+ & f_4^+ \\ (0.3,0.4,0.5) & (0.7,0.8,0.9) & (0.7,0.8,0.9) & (0.6,0.7,0.8) \end{Bmatrix}$$

$$A^- = \begin{Bmatrix} f_1^- & f_2^- & f_3^- & f_4^- \\ (0.1,0.2,0.3) & (0.2,0.4,0.5) & (0.2,0.3,0.4) & (0.1,0.3,0.4) \end{Bmatrix}$$

Step-2: Calculate the separation measures using the hesitant normalized Hamming distance. The separation of each alternative from the ideal is given as:

$$D_1^+ = \|A_1 - A^+\| = \sum_{j=1}^{4} \|f_{ij} - f_j^+\| = \|f_{11} - f_1^+\| + \|f_{12} - f_2^+\| + \|f_{13} - f_3^+\| + \|f_{14} - f_4^+\| = 1.5$$

$$D_2^+ = \|A_2 - A^+\| = \sum_{j=1}^{4} \|f_{2j} - f_j^+\| = \|f_{21} - f_1^+\| + \|f_{22} - f_2^+\| + \|f_{23} - f_3^+\| + \|f_{24} - f_4^+\| = 2.4$$

Similarly $D_3^+ = 1.95$ and $D_4^+ = 3.6$

The separation from the negative ideal solution is given as

$$D_1^- = \left\| A_1 - A^- \right\| = \sum_{j=1}^{4} \left\| f_{ij} - f_j^- \right\| = 2.95$$

$D_2^- = 2.3$; $D_3^- = 2.6$; $D_4^- = 1.3$

Step-3: Calculate the relative closeness to the ideal solution by the equation $D_i = \dfrac{D^-_i}{D_i^- + D_i^+}$

$$D_1 = 0.6629; D_2 = 0.4894; \quad D_3 = 0.5714 \text{ and } D_4 = 0.265$$

Step-4: Ranking the preference order we get $D_1 > D_3 > D_2 > D_4$. Hence A_1 is the best alternative.

6. CONCLUSIONS

This paper examines two popular multi-criteria decision making algorithms such as fuzzy VIKOR and fuzzy TOPSIS using triangular hesitant fuzzy sets. The hesitant fuzzy environment when applied is very helpful in decision making situations when an expert might consider multiple priorities or judgments. All the MCDM methods estimate criteria weights so that human judgment can be avoided by assigning weights to different attributes. Both the methods result in same preference of selecting a teacher based on Quality evaluation. But fuzzy VIKOR method stands out to be the best due to its computational easiness.

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